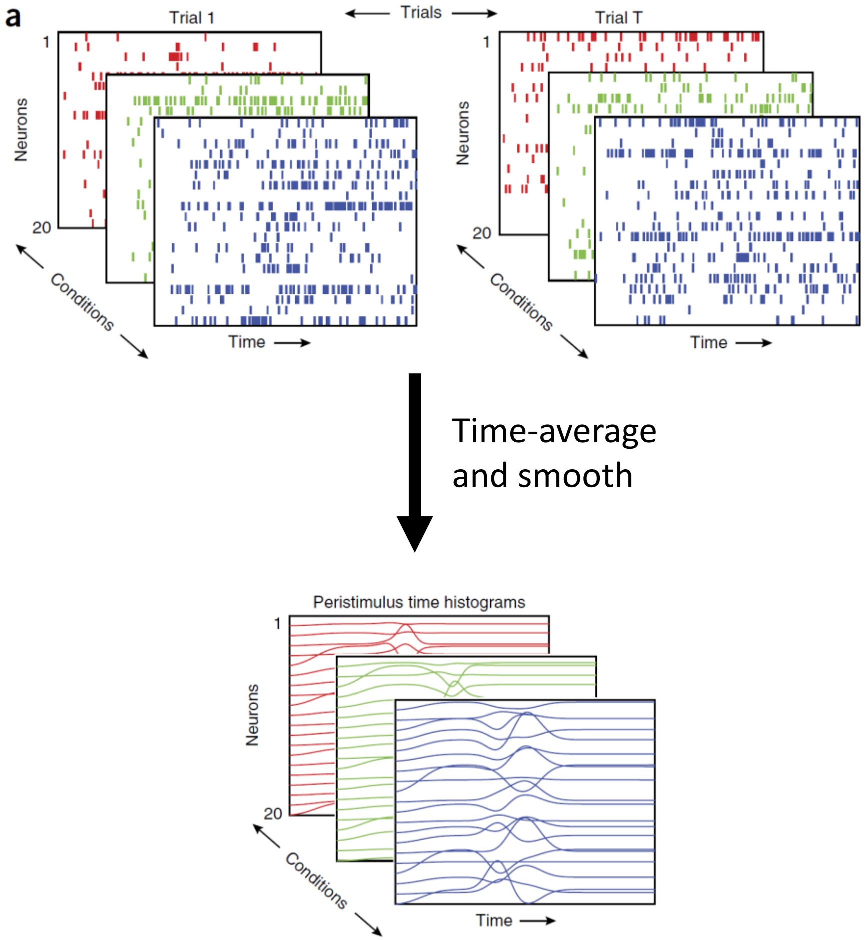
# Recovering Behavioral Dynamics with Gaussian Process Factor Analysis

Mason del Rosario – MAE 298 – 2019-June-06

# Background: Dimensional Reduction

- Neural population datasets have grown in size
- Dimensional Reduction
  1. Retain statistical power of individual trials
  2. Analysis of population-level response structure
  3. Facilitate exploratory data analysis (EDA)




Cunningham, M., and Yu, B., "Dimensionality reduction for large-scale neural recordings," *Nature Neuroscience*, Aug 2014.

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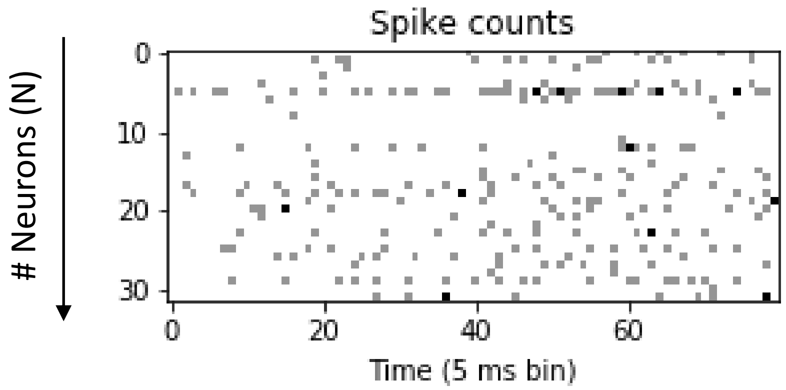


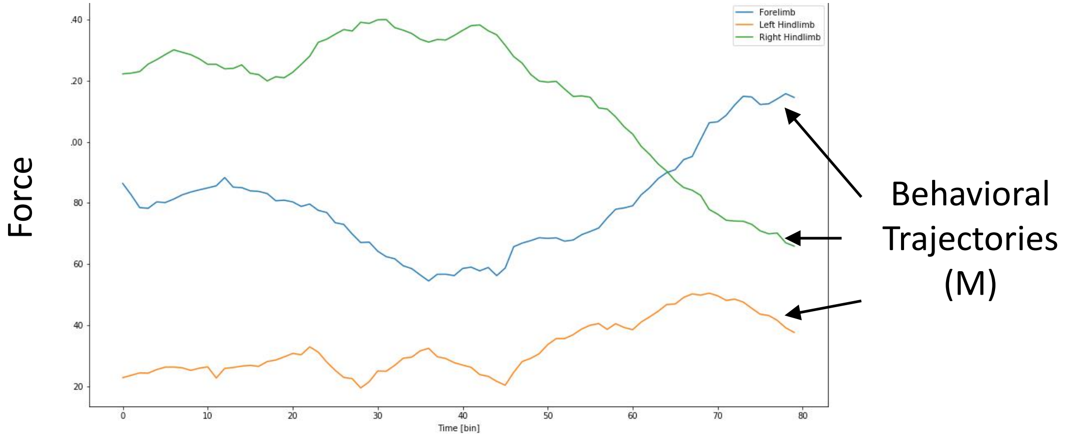


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# Background: PCA vs. LFA

**Given a set of observations**

$$\{X_1, X_2, \ldots, X_p\}$$

**Principal Component Analysis (PCA)**

$$PC_1 = a_1 X_1 + \cdots + a_p X_p$$
$$PC_2 = b_1 X_1 + \cdots + b_p X_p$$
$$\vdots$$
$$PC_n = z_1 X_1 + \cdots + z_p X_p$$

Seeks to find PCs/subspace which explains variance in data; *model-free*

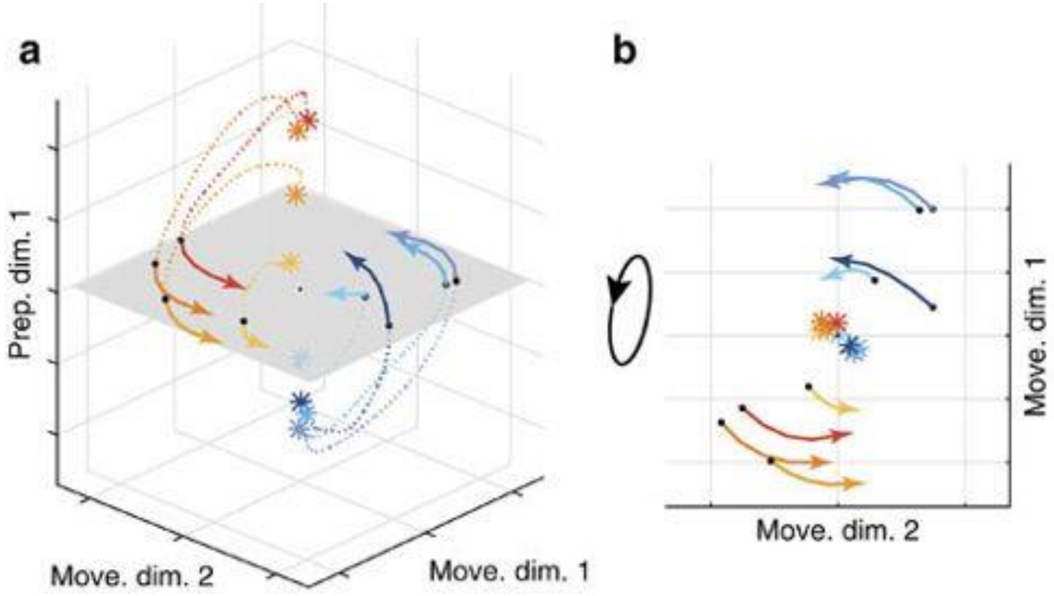
**Latent Factor Analysis (LFA)**

$$X_1 = \alpha_1 f_1 + \cdots + \alpha_m f_m + w_1$$
$$X_2 = \beta_1 f_1 + \cdots + \beta_m f_m + w_2$$
$$\vdots$$
$$X_p = \gamma_1 f_1 + \cdots + \gamma_p f_m + w_m$$

Seeks to explain data assuming underlying dynamics; *model-based*

# Background: Latent vs. Behavioral Dynamics

- Latent trajectories converge based on event
- Do latent factor trajectories have temporal correlation with behavioral signals of interest?




Elsayad, G. F., Lara, A. H., Kaufman, M. T., Churchland, M. M., Cunningham, J. P., "Reorganization between preparatory and movement population responses in motor cortex," Nature Communications 7, 13239, Oct. 2016.

# Goals

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1. Optimize hyperparameters (i.e. input bin size, dimensionality of latent space) to minimize spike count prediction error.
2. Use GPFA (a form of LFA) on tilt data to visualize latent trajectories.

# Goals/Hypotheses

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1. Optimize hyperparameters (i.e. input bin size, dimensionality of latent space) to minimize spike count prediction error.
2. Use GPFA (a form of LFA) on tilt data to visualize latent trajectories.

**Hypotheses**

1. Hyperparameters
   1. Bin size should be small to capture timing information
   2. Dimensionality should be similar to that of behavioral data
2. Latent trajectories
   1. Should converge to common subspace
   2. Should explain behavioral data

# Methods: GPFA

- Gaussian Process Factor Analysis (GPFA) is a form of Latent Factor Analysis
- Latent variables evolve as a Gaussian process
- Goal: find model parameters (e.g., covariance matrix)
- Python implementation by Macke ([Github]())

**Gaussian Process Factor Analysis (GPFA)**

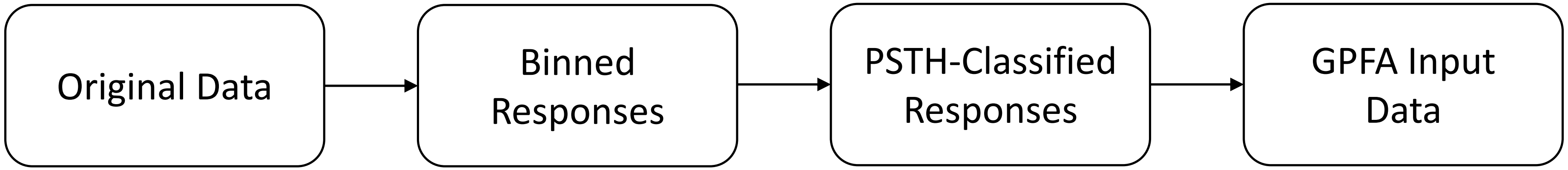
$$X_{1,t} = \alpha_1 f_{1,t} + \cdots + \alpha_m f_{m,t} + w_{1,t}$$
$$X_{2,t} = \beta_1 f_{1,t} + \cdots + \beta_m f_{m,t} + w_{2,t}$$
$$\vdots$$
$$X_{p,t} = \gamma_1 f_{1,t} + \cdots + \gamma_p f_{m,t} + w_{m,t}$$

$$\mathbf{f}_t = \begin{bmatrix} f_{1,t} & \ldots & f_{m,t} \end{bmatrix}^T$$

$$\mathbf{f}_1 \sim \mathcal{N}(\mathbf{f}_0, Q_0)$$
$$\mathbf{f}_t | \mathbf{f}_{t-1} \sim \mathcal{N}(A\mathbf{f}_{t-1} + B\mathbf{u}_t, Q)$$

J. H. Macke, L. Buesing, and M. Sahani, “Estimating state and parameters in state space models of spike trains,” January 2015

# GPFA: Data Workflow



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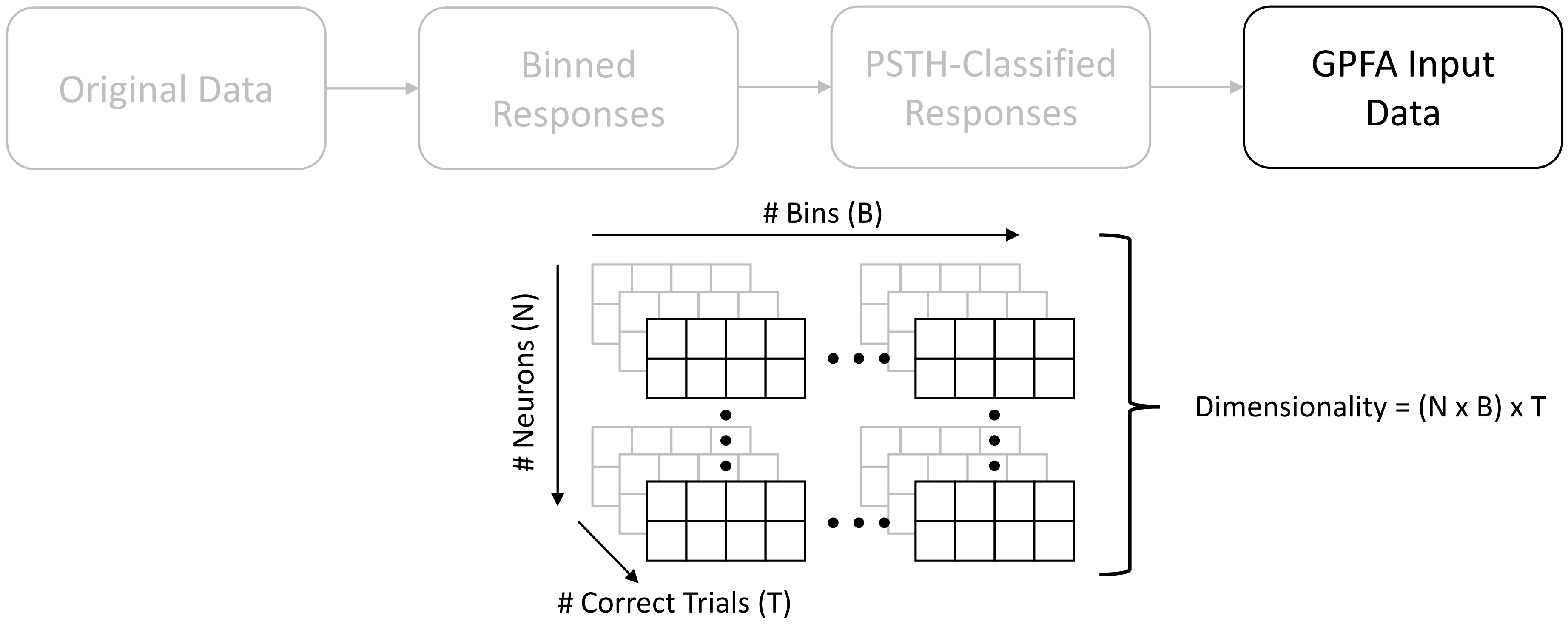
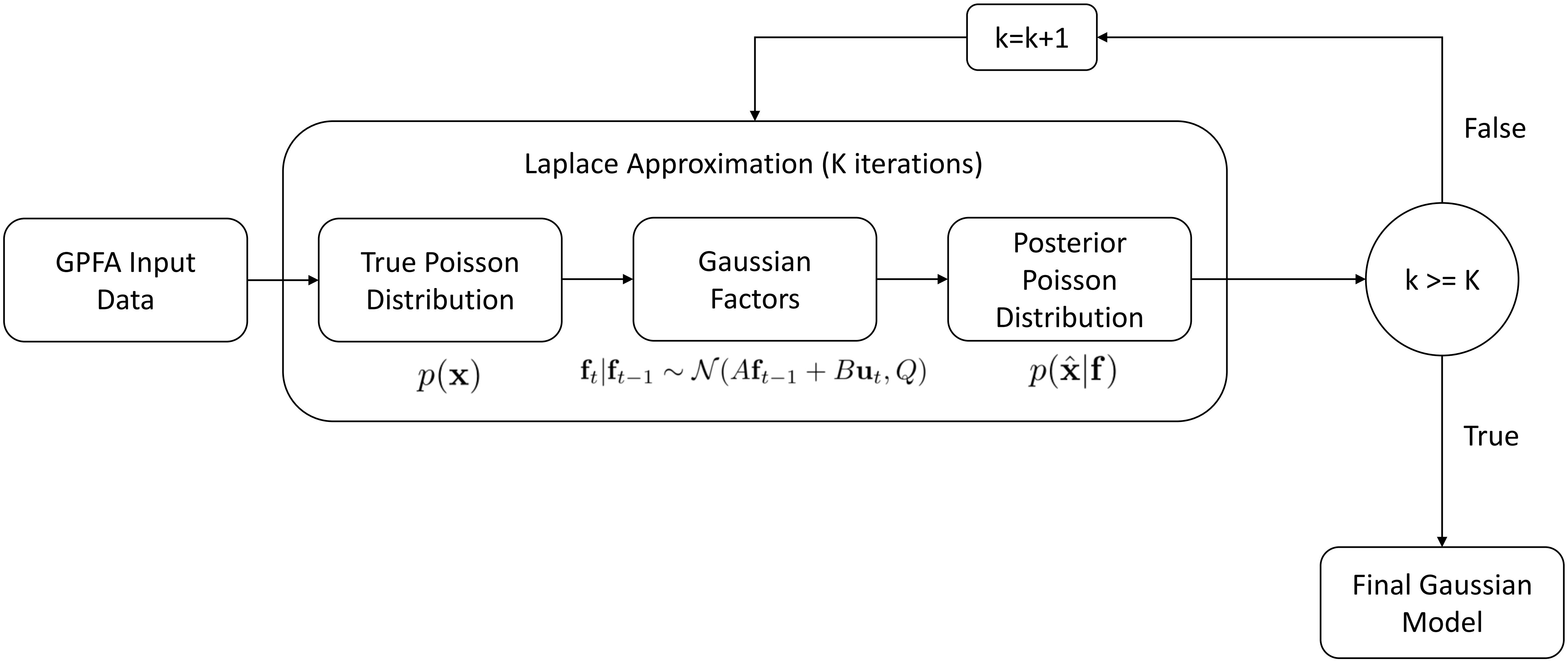


# GPFA: Parameter Estimation

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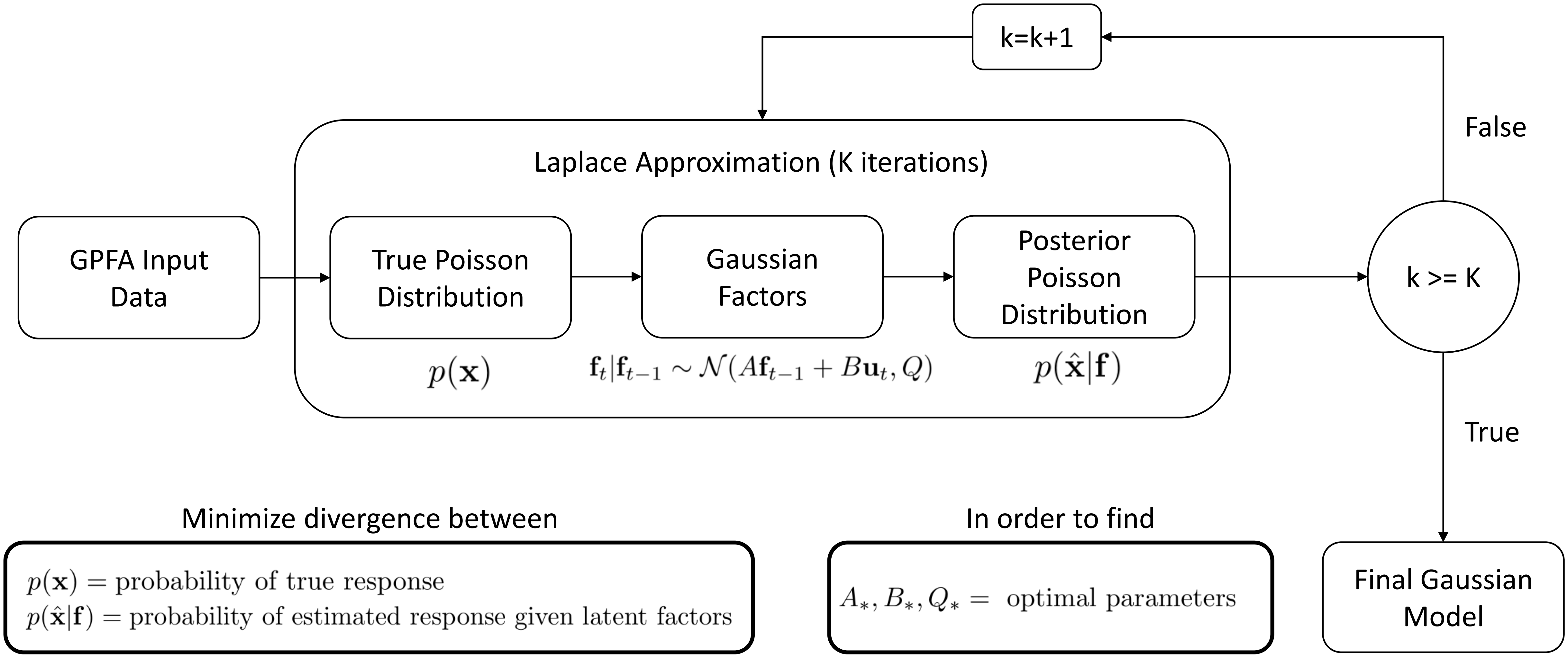
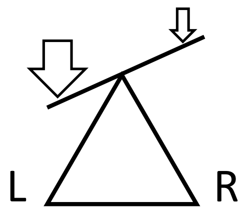
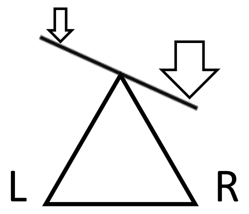


# Results: Tilt Task

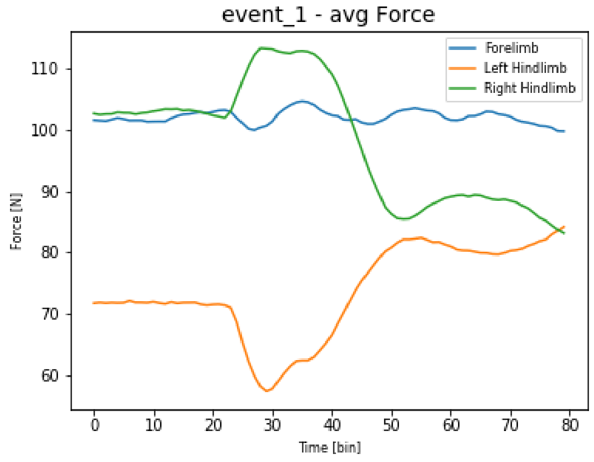
| Event | Left | Right |
|---|---|---|
| event_1 | Extension | Flexion |
| event_3 | Extension | Flexion |
| event_4 | Flexion | Extension |
| event_6 | Flexion | Extension |

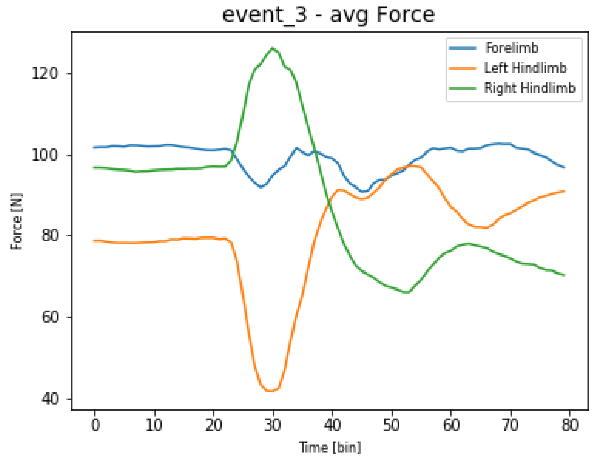


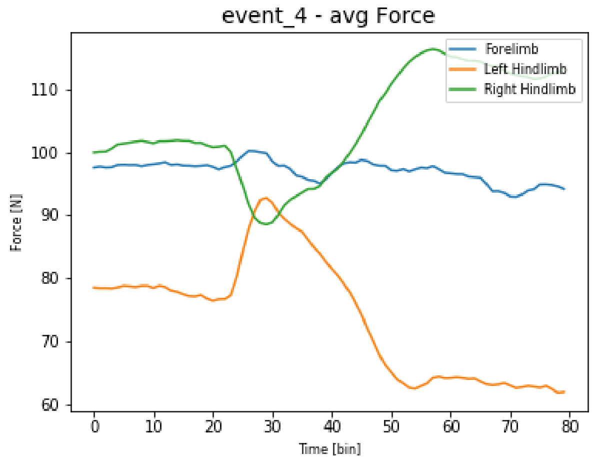

event_1, event_3

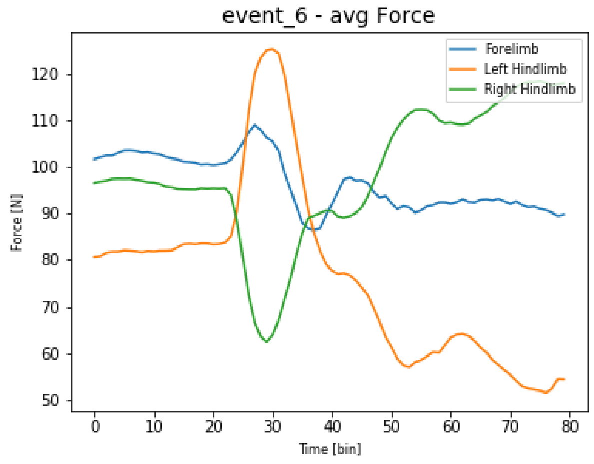



event_4, event_6



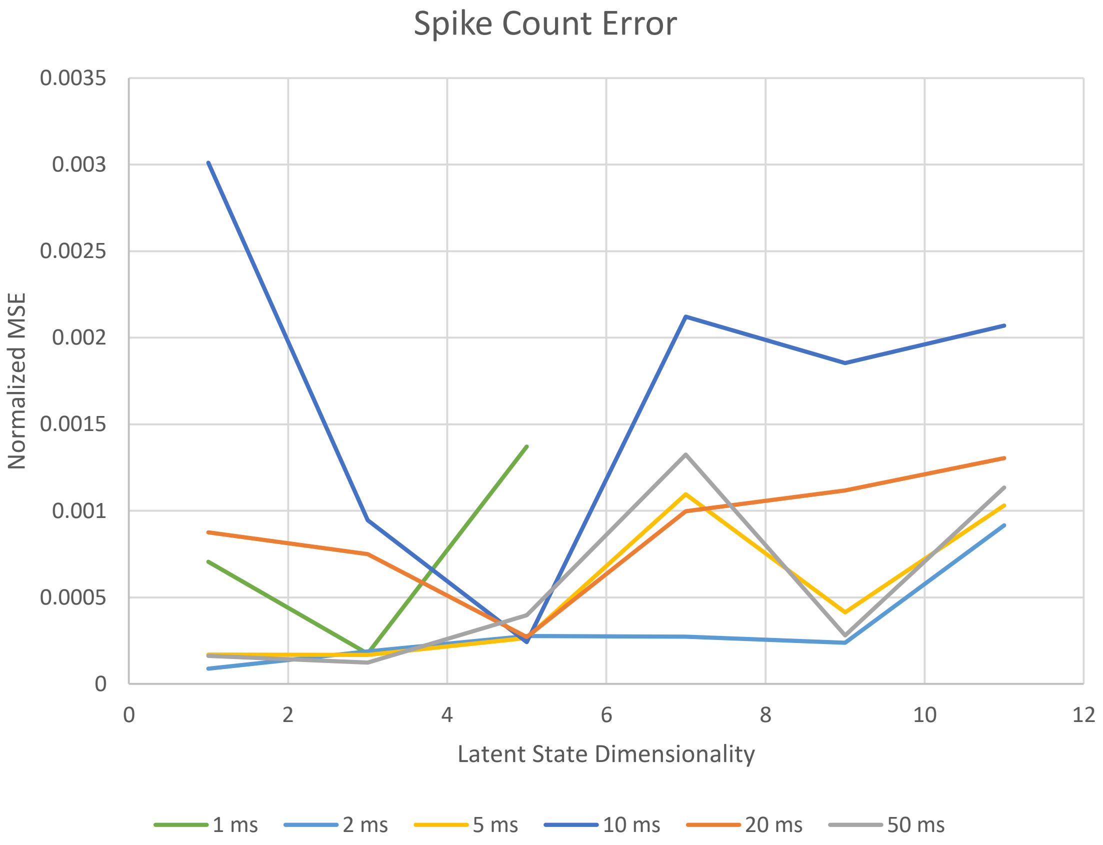



# Results: Hyperparameter Optimization

| Hyperparameter | Optimal Range |
|---|---|
| Bin Size | 2 to 5ms |
| Latent Dimensionality | 3 to 5 |

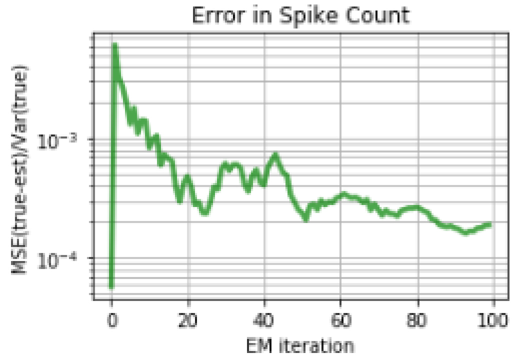
**Normalized Mean-square Error**

$$E_{\text{pred}} = \frac{\mathrm{E}\left[(\hat{X} - X)^2\right]}{\sigma_X^2}$$

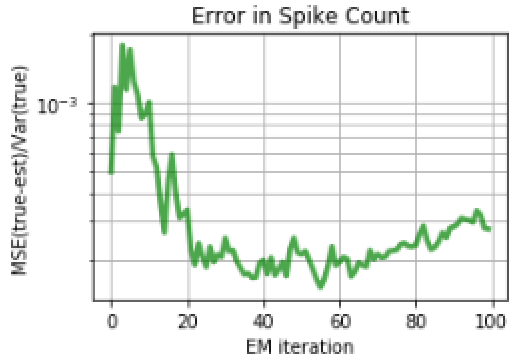


# Discussion: Hyperparameter Optimization

- Lower bin size (i.e., 2ms) yields lower prediction error
  - Computationally prohibitive O(T^2)
- Used ‘Online’ method; subsampling of trials
  - Stochasticity in prediction/noise in error



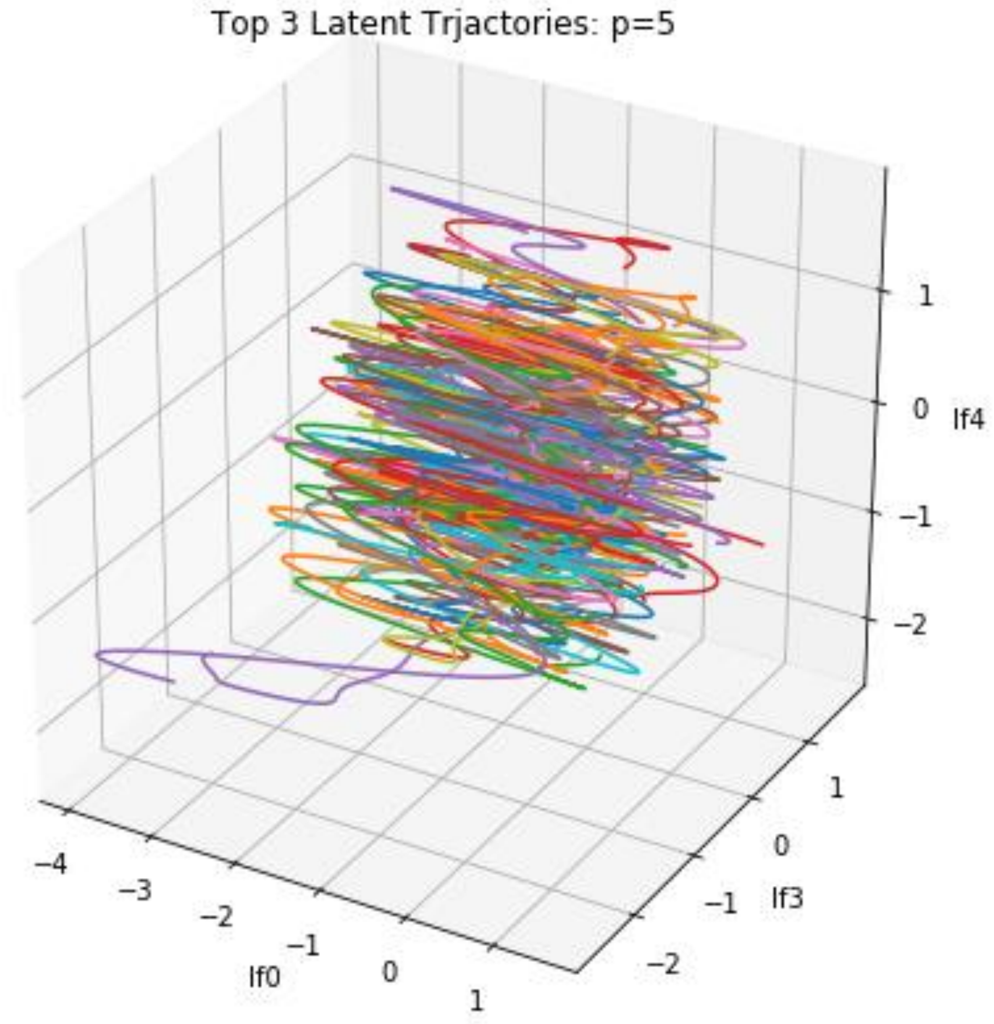

Bin Size: 2ms - Latent Dimensionality: 3




Bin Size: 2ms - Latent Dimensionality: 5

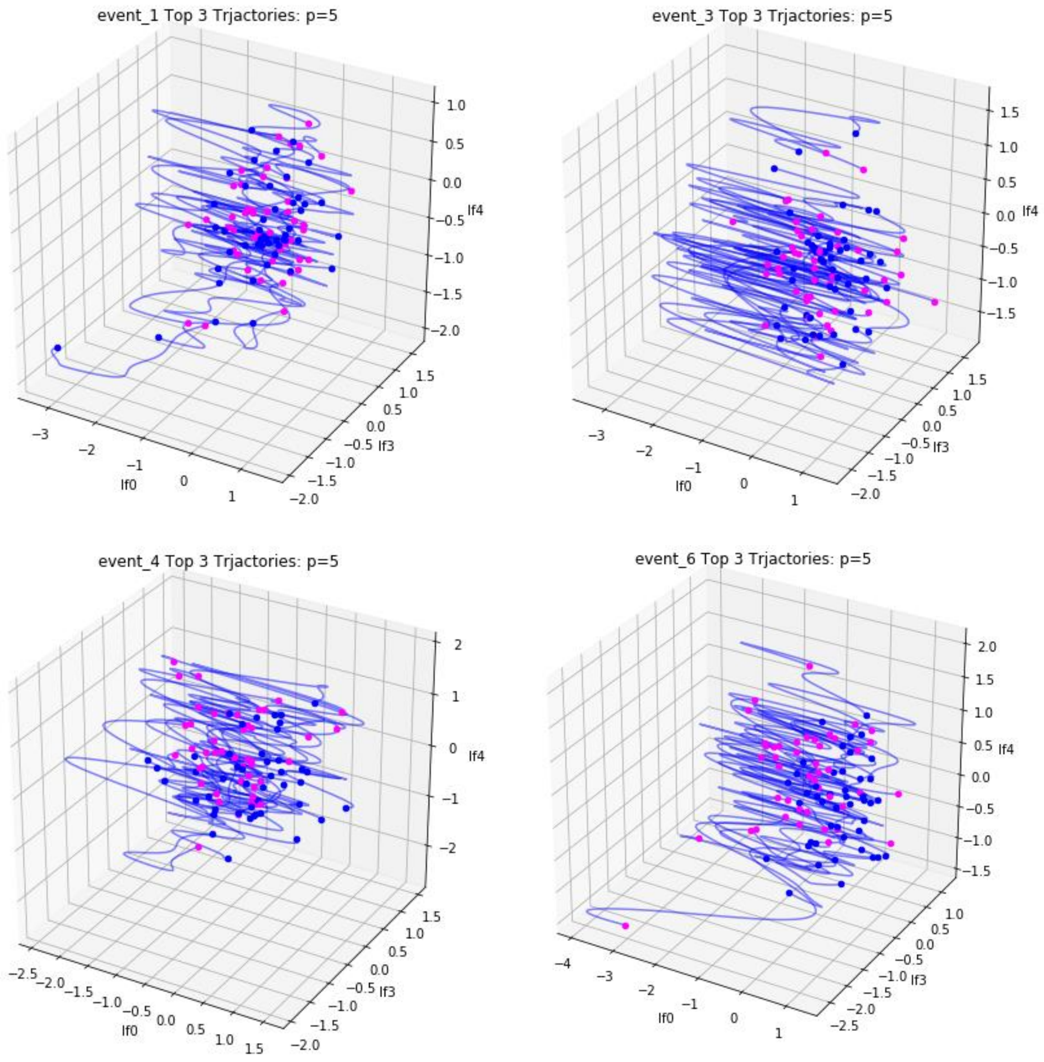
# Results: Latent Trajectories

- Bin size=5ms
- Highest variance trajectories
- Unclear as to whether these converge to a planar subspace

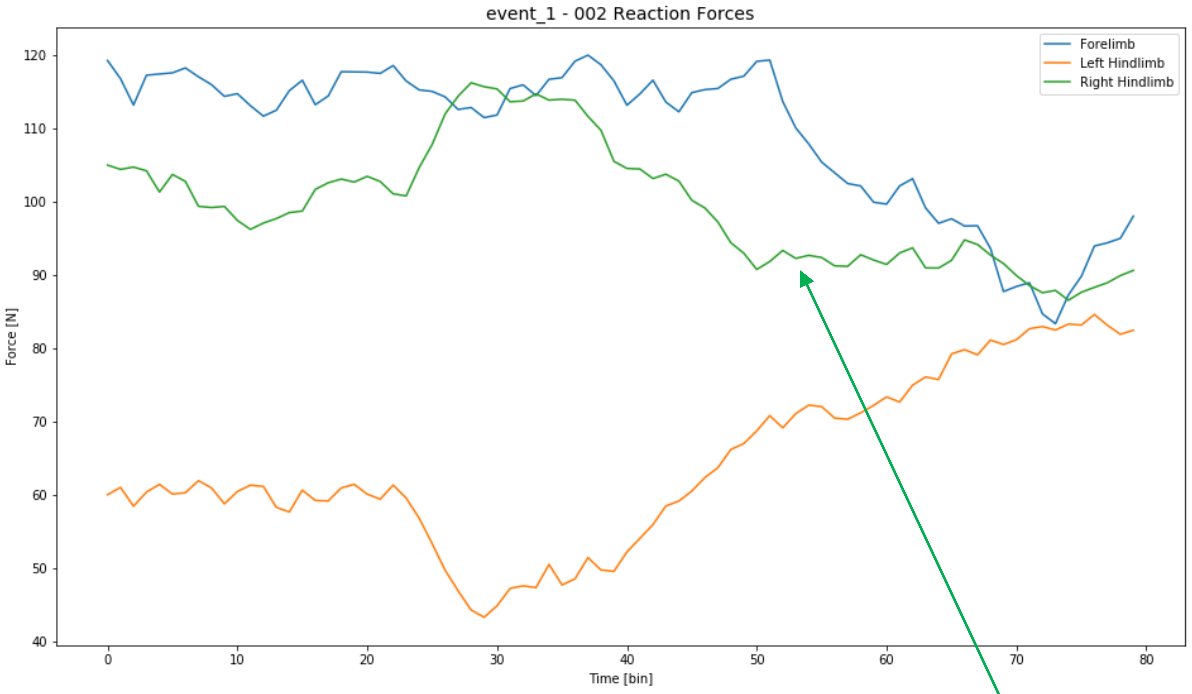


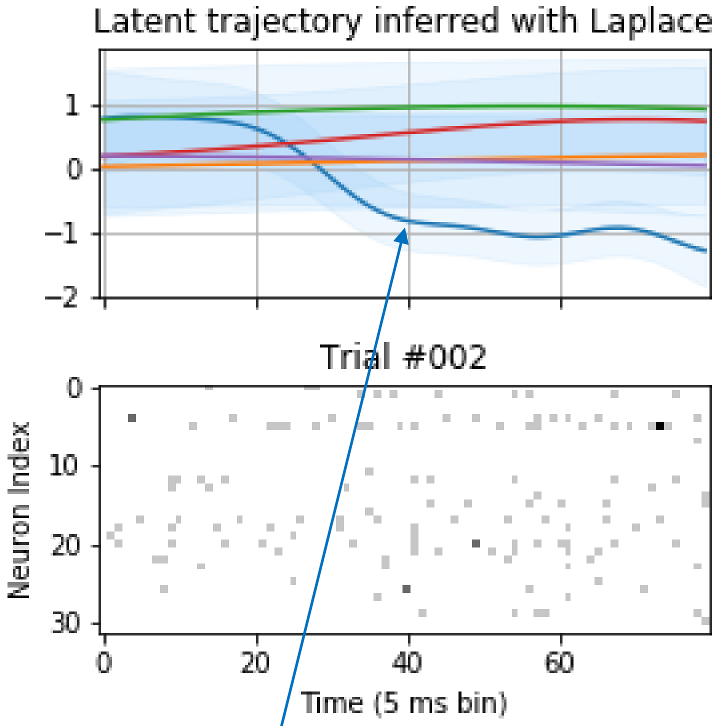
# Results: Event-sorted Trajectories

- Correct trials/per event
- Trajectory endpoints (blue = start, purple = end)
- Planar subspace still inconclusive



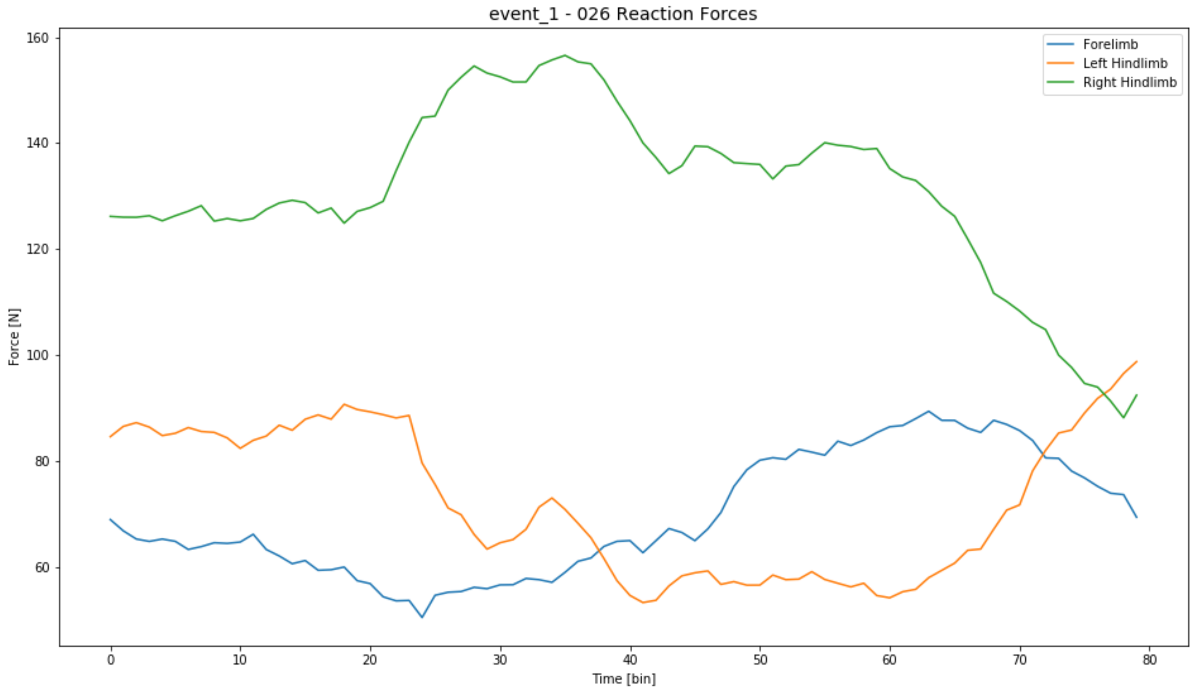
# Event_1: Behavioral vs. Latent Trajectories

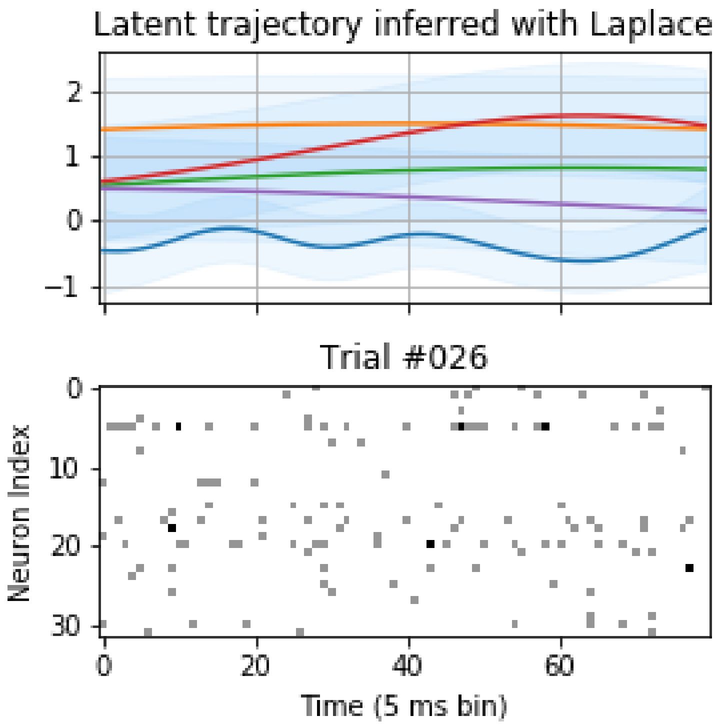



**Positive** temporal correlation between right hindlimb force and latent blue trajectory.
Right hindlimb flexion encoded in blue latent factor? Expect left hemisphere neurons to fire.

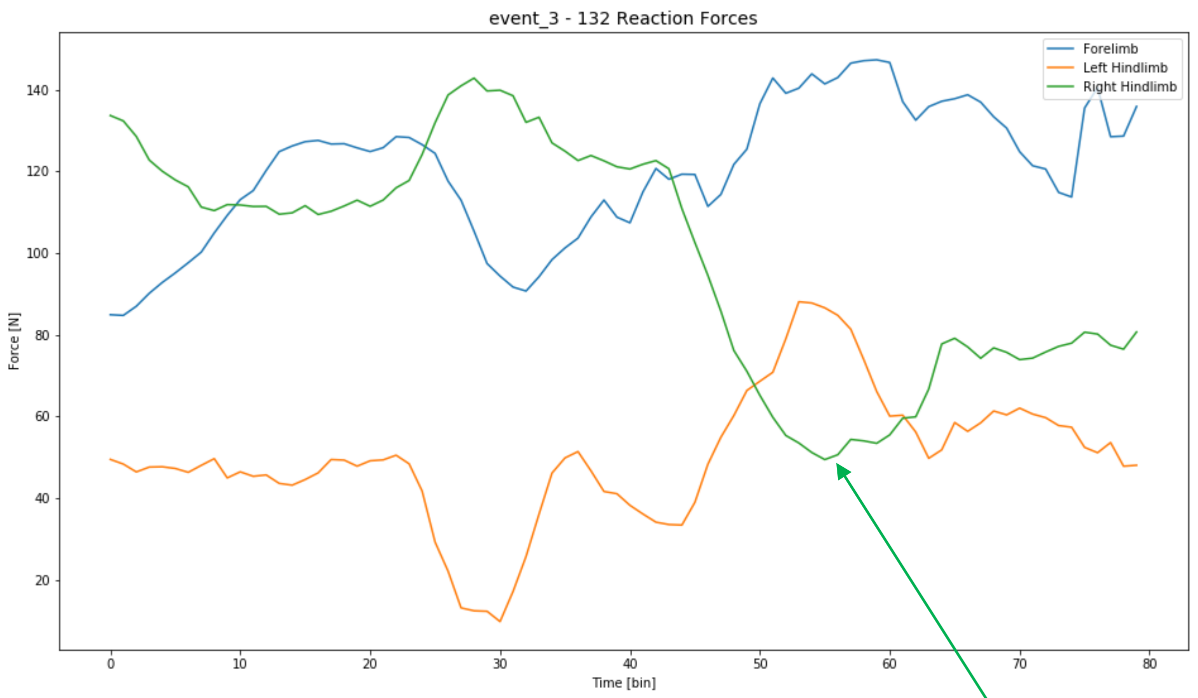
# Event_1: Behavioral vs. Latent Trajectories

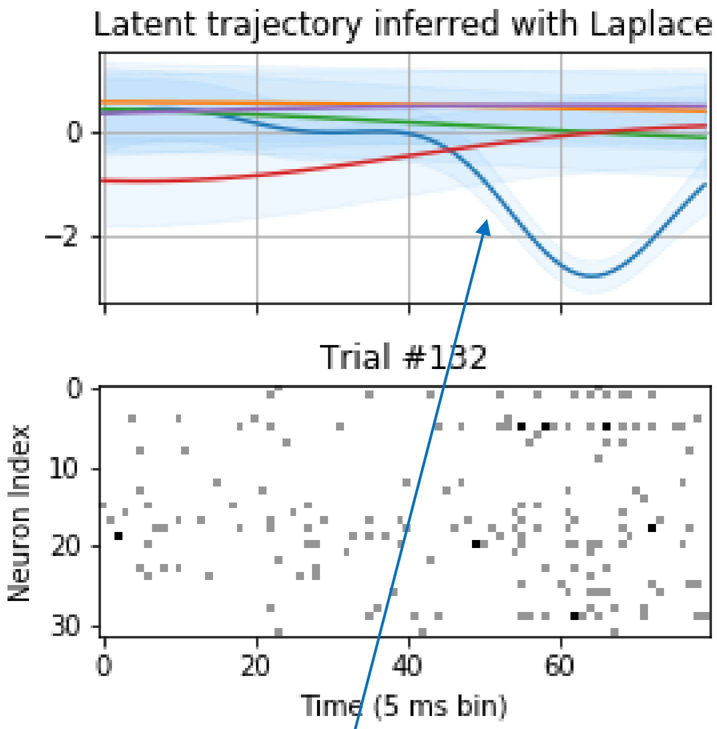



**Correlation less obvious…**

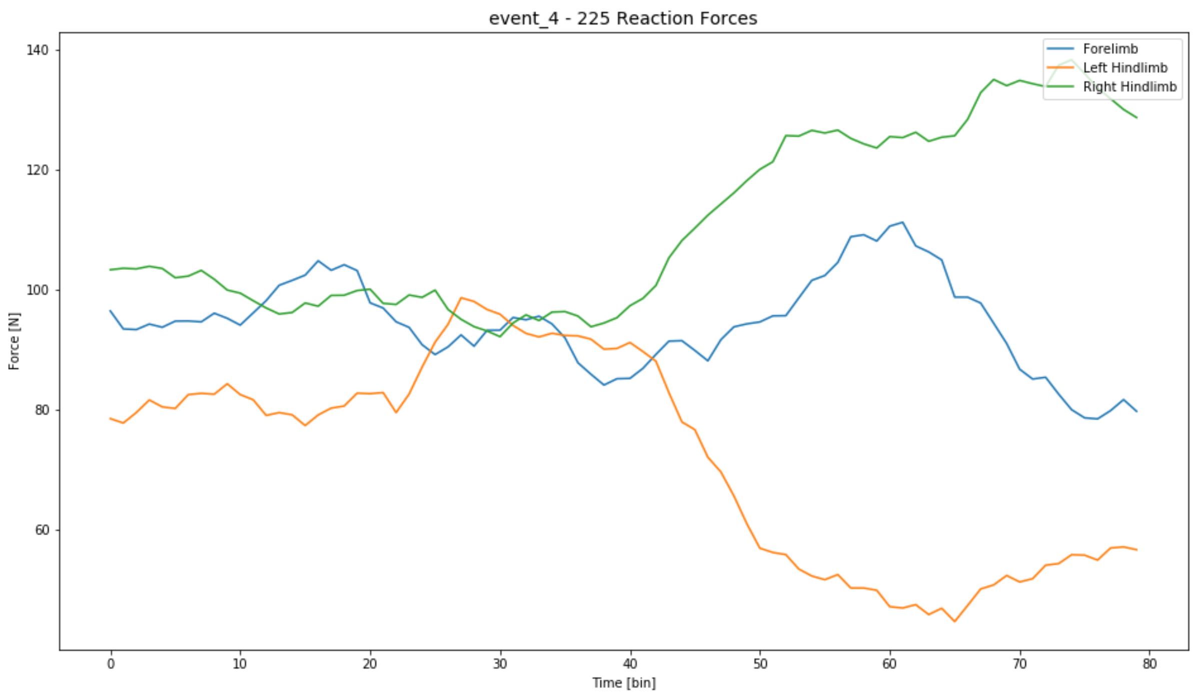
# Event_3: Behavioral vs. Latent Trajectories

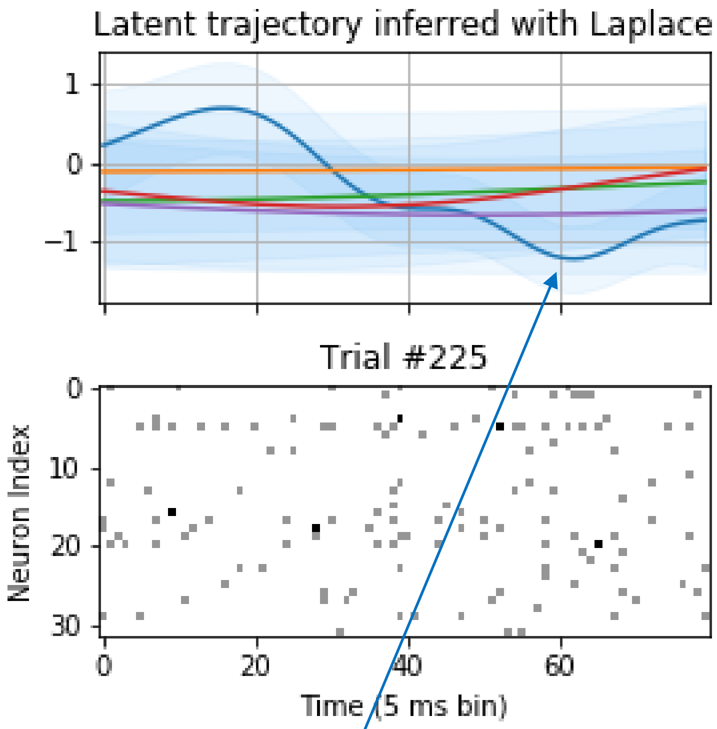



**Positive** temporal correlation between behavioral green trajectory and latent blue trajectory.

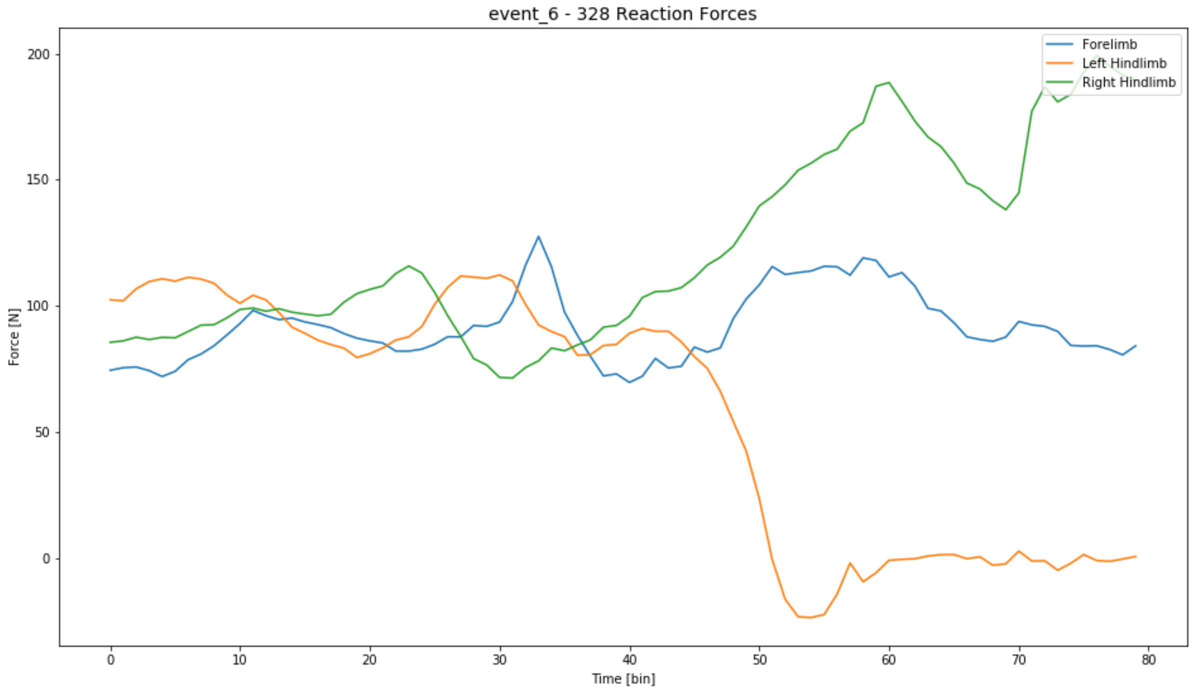
# Event_4: Behavioral vs. Latent Trajectories

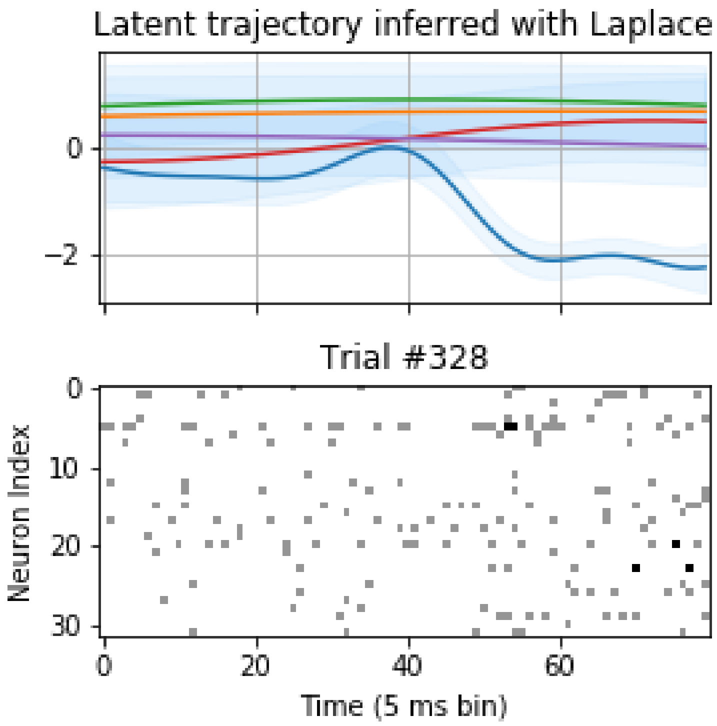



**Negative** temporal correlation between right hindlimb force and latent blue trajectory?
**Positive** temporal correlation between left hindlimb force and latent blue trajectory?

# Event_6: Behavioral vs. Latent Trajectories




**Negative** temporal correlation between right hindlimb force and latent blue trajectory?
**Positive** temporal correlation between left hindlimb force and latent blue trajectory?

# Conclusions/Future Work

- **Hyperparameters**
  - 5ms, p=5 are optimal
  - **Noise** – higher batch count; more compute?
- **Latent factors**
  - Planar subspace convergence = not evident
    - Try more/different latent dimensionality?
  - Single factor loading
  - Correlation with behavioral data = not apparent
    - More quantitative comparison (e.g. cross-correlation, optimal linear estimation)
    - Spike rate-based optimization
- **Methods**
  - Other estimation methods?
    - Gaussian variational inference

# Questions?